Rich Behavioral Models: Illustration on Journey Planning

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Verification seminar — University of Oxford

The talk in one slide

Strategy synthesis for Markov Decision Processes (MDPs)

Finding **good** controllers for systems interacting with a *stochastic* environment.

- Good? Performance evaluated through *payoff functions*.
- Usual problem is to optimize the *expected performance* or the *probability of achieving a given performance level*.
- Not sufficient for many practical applications.
  - ▷ Several extensions, more expressive but also more complex...

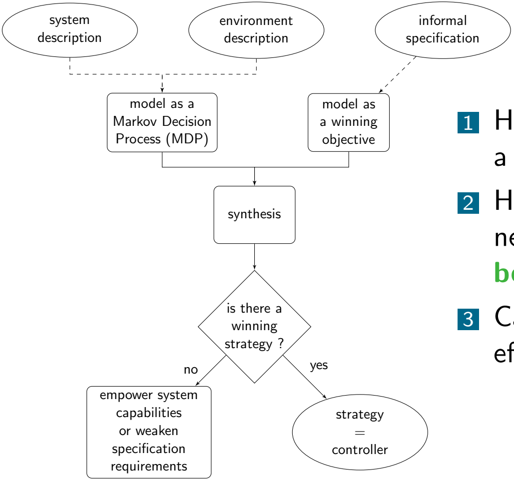
Aim of this survey talk

Give a flavor of classical questions and extensions (*rich behavioral models*), illustrated on the stochastic shortest path (SSP).

Multi-criteria quantitative synthesis

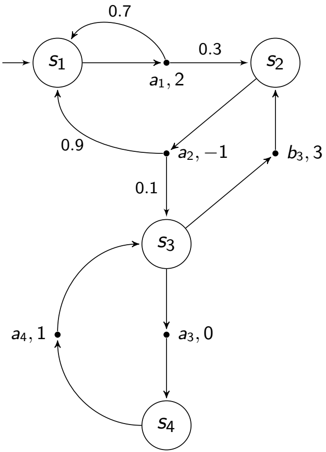
- Verification and synthesis:
  - a reactive **system** to *control*,
  - an *interacting* **environment**,
  - a **specification** to *enforce*.
- Model of the (discrete) interaction?
  - Antagonistic environment: 2-player game on graph.
  - **Stochastic environment: MDP**.
- **Quantitative** specifications. Examples:
  - Reach a state s before x time units $\rightsquigarrow$ shortest path.
  - Minimize the average response-time $\rightsquigarrow$ mean-payoff.
- Focus on **multi-criteria quantitative models**
  - to reason about *trade-offs* and *interplays*.

Strategy (policy) synthesis for MDPs



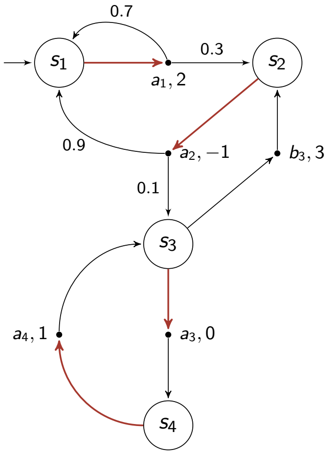
1. How complex is it to decide if a winning strategy exists?
2. How complex such a strategy needs to be? **Simpler is better**.
3. Can we synthesize one efficiently?

Markov decision processes



- MDP $D = (S, s_{\mathsf{init}}, A, \delta, w)$.
  - ▷ Finite sets of states S and actions A,
  - ▷ probabilistic transition $\delta\colon S \times A \to \mathcal{D}(S)$,
  - ▷ weight function $w\colon A \to \mathbb{Z}$.
- **Run** (or play): $\rho = s_1 a_1 \dots a_{n-1} s_n \dots$ such that $\delta(s_i, a_i, s_{i+1}) > 0$ for all $i \geq 1$.
  - ▷ Set of runs $\mathcal{R}(D)$.
  - ▷ Set of histories (finite runs) $\mathcal{H}(D)$.
- **Strategy** $\sigma\colon \mathcal{H}(D) \to \mathcal{D}(A)$.
  - ▷ $\forall\, h$ ending in s, $\mathsf{Supp}(\sigma(h)) \in A(s)$.

Markov decision processes



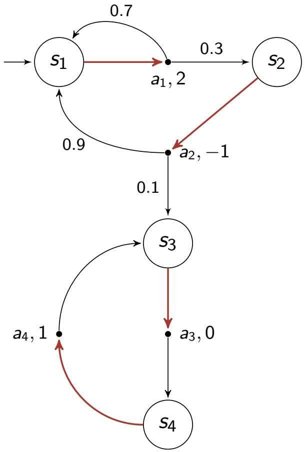
Sample *pure memoryless* strategy σ.

Sample run $\rho = s_1 a_1 s_2 a_2 s_1 a_1 s_2 a_2 (s_3 a_3 s_4 a_4)^\omega$.

Other possible run $\rho' = s_1 a_1 s_2 a_2 (s_3 a_3 s_4 a_4)^\omega$.

- Strategies may use
  - finite or infinite **memory**,
  - **randomness**.
- **Payoff functions** map runs to numerical values:
  - truncated sum up to $T = \{s_3\}$: $\mathsf{TS}^T(\rho) = 2$, $\mathsf{TS}^T(\rho') = 1$,
  - mean-payoff: $\underline{\mathsf{MP}}(\rho) = \underline{\mathsf{MP}}(\rho') = 1/2$,
  - many more.

Markov chains



Once strategy σ fixed, fully stochastic process:

$\rightsquigarrow$ **Markov chain** (**MC**) M.

State space = product of the MDP and the memory of σ.

- Event $\mathcal{E} \subseteq \mathcal{R}(M)$
  - ▷ probability $\mathbb{P}_M(\mathcal{E})$
- Measurable $f : \mathcal{R}(M) \to \mathbb{R} \cup \{\infty\}$,
  - ▷ expected value $\mathbb{E}_M(f)$

Aim of this survey

Compare different types of quantitative specifications for MDPs

- ▷ w.r.t. the complexity of the decision problem,
- ▷ w.r.t. the complexity of winning strategies.

Recent extensions share a common philosophy: framework for the synthesis of strategies with *richer performance guarantees*.

- ▷ Our work deals with many different payoff functions.

Focus on the **shortest path problem** in this talk.

- ▷ Not the most involved technically, natural applications.
- ⇝ Useful to understand the practical interest of each variant.

Joint work with R. Berthon, V. Bruyère, E. Filiot, J.-F. Raskin, O. Sankur [BFRR17, RRS17, RRS15, BCH+16, Ran16, BRR17].

Stochastic shortest path

Shortest path problem for *weighted graphs*

Given state $s \in S$ and target set $T \subseteq S$, find a path from s to a state $t \in T$ that minimizes the sum of weights along edges.

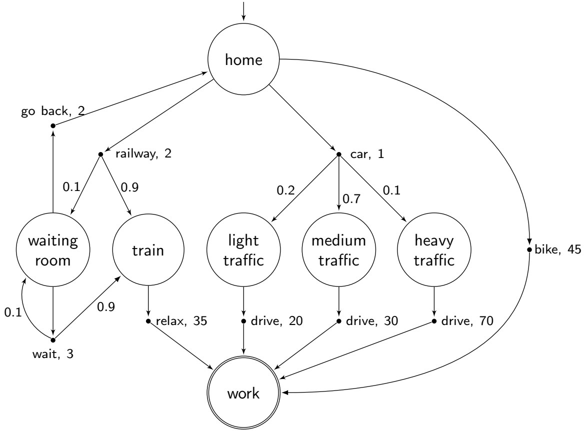
- ▷ PTIME algorithms (Dijkstra, Bellman-Ford, etc) [CGR96].

We focus on MDPs with **strictly positive weights** for the SSP.

- ▷ **Truncated sum** payoff function for $\rho = s_1 a_1 s_2 a_2 \ldots$ and target set T:

$$\mathsf{TS}^T(\rho) = \begin{cases} \sum_{j=1}^{n-1} w(a_j) \text{ if } s_n \text{ first visit of } T, \\ \infty \text{ if } T \text{ is never reached.} \end{cases}$$

Planning a journey in an uncertain environment



Each action takes time, target = work.

- ▷ What kind of strategies are we looking for when the environment is stochastic?

SSP-E: minimizing the expected length to target

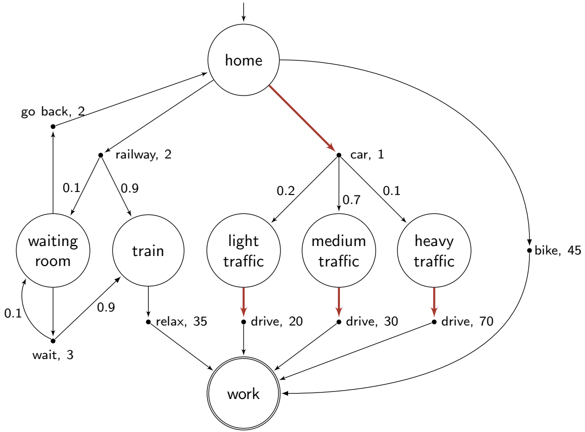
SSP-E problem

Given MDP $D = (S, s_{\text{init}}, A, \delta, w)$, target set T and threshold $\ell \in \mathbb{Q}$, decide if there exists σ such that $\mathbb{E}^{\sigma}_{D}(\mathsf{TS}^{T}) \leq \ell$.

Theorem [BT91]

The SSP-E problem can be decided in polynomial time. Optimal pure memoryless strategies always exist and can be constructed in polynomial time.

SSP-E: illustration



- ▷ Pure memoryless strategies suffice.
- ▷ Taking the **car** is optimal: $\mathbb{E}_D^\sigma(\mathsf{TS}^T) = 33$.

SSP-E: PTIME algorithm

1. Graph analysis (linear time):
   - s not connected to $T \Rightarrow \infty$ and remove,
   - $s \in T \Rightarrow 0$.
2. **Linear programming** (**LP**, polynomial time).

For each $s \in S \setminus T$, one variable x_s,

$$\max \sum_{s \in S \setminus T} x_s$$

under the constraints

$$x_s \leq w(a) + \sum_{s' \in S \setminus T} \delta(s, a, s') \cdot x_{s'} \qquad \text{for all } s \in S \setminus T, \text{ for all } a \in A(s).$$

SSP-E: PTIME algorithm

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   - s not connected to $T \Rightarrow \infty$ and remove,
   - $s \in T \Rightarrow 0$.
2. **Linear programming** (**LP**, polynomial time).

Optimal solution $\mathbf{v}$:

$\rightsquigarrow$ $\mathbf{v}_s =$ expectation from s to T under an optimal strategy.

Optimal pure memoryless strategy $\sigma^{\mathbf{v}}$:

$$\sigma^{\mathbf{v}}(s) = \arg \min_{a \in A(s)} \left[w(a) + \sum_{s' \in S \setminus T} \delta(s, a, s') \cdot \mathbf{v}_{s'} \right].$$

$\rightsquigarrow$ **Playing optimally = locally optimizing present + future.**

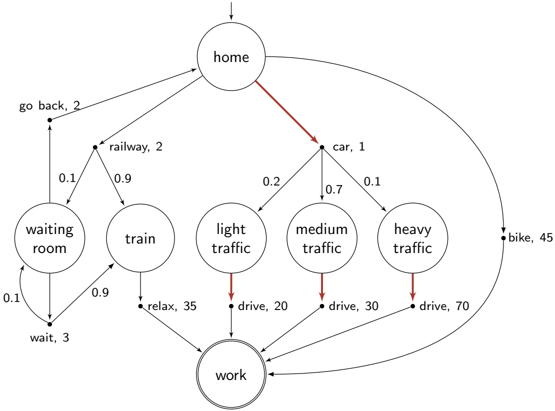
SSP-E: PTIME algorithm

1. Graph analysis (linear time):
   - s not connected to $T \Rightarrow \infty$ and remove,
   - $s \in T \Rightarrow 0$.
2. **Linear programming** (**LP**, polynomial time).

In practice, **value and strategy iteration** algorithms often used:

- best performance in most cases but exponential in the worst-case,
- fixed point algorithms, successive solution improvements [BT91, dA99, HM14].

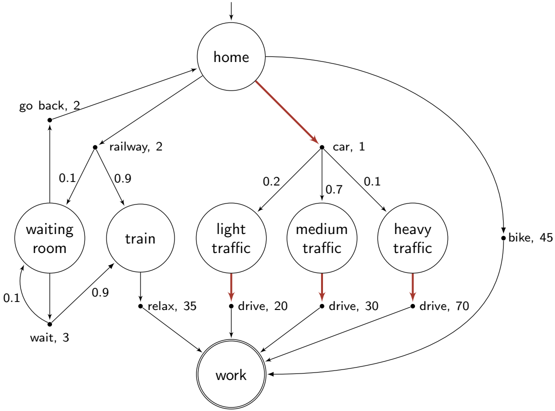
Traveling without taking too many risks



Minimizing the *expected time* to destination makes sense **if** we travel often and it is not a problem to be late.

With car, in 10% of the cases, the journey takes 71 minutes.

Traveling without taking too many risks



Most bosses will not be happy if we are late too often...

$\rightsquigarrow$ what if we are risk-averse and want to avoid that?

SSP-P: forcing short paths with high probability

SSP-P problem

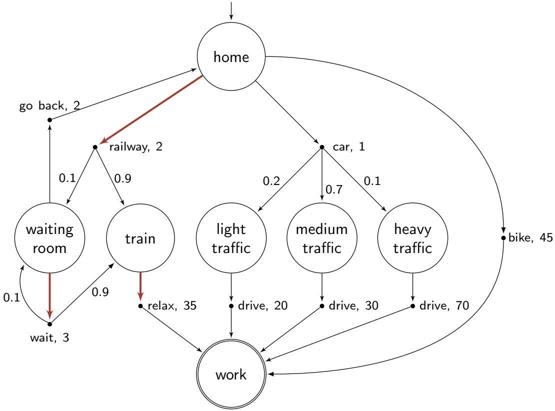
Given MDP $D = (S, s_{\text{init}}, A, \delta, w)$, target set T, threshold $\ell \in \mathbb{N}$, and probability threshold $\alpha \in [0, 1] \cap \mathbb{Q}$, decide if there exists a strategy σ such that $\mathbb{P}^{\sigma}_{D}\big[\{\rho \in \mathcal{R}_{s_{\text{init}}}(D) \mid \mathsf{TS}^{T}(\rho) \leq \ell\}\big] \geq \alpha$.

Theorem

The SSP-P problem can be decided in pseudo-polynomial time, and it is PSPACE-hard. Optimal pure strategies with pseudo-polynomial memory always exist and can be constructed in pseudo-polynomial time.

See [HK15] for hardness and for example [RRS17] for algorithm.

SSP-P: illustration



Specification: reach work within 40 minutes with 0.95 probability

Sample strategy: take the **train** $\rightsquigarrow \mathbb{P}^{\sigma}_{D}\big[\mathsf{TS}^{\mathsf{work}} \leq 40\big] = 0.99$

Bad choices: car (0.9) and bike (0.0)

SSP-P: pseudo-PTIME algorithm (1/2)

Key idea: pseudo-PTIME reduction to the **stochastic reachability problem** (**SR**)

SR problem

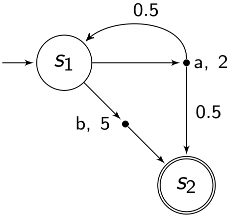
Given unweighted MDP $D = (S, s_{\text{init}}, A, \delta)$, target set T and probability threshold $\alpha \in [0, 1] \cap \mathbb{Q}$, decide if there exists a strategy σ such that $\mathbb{P}^{\sigma}_{D}\left[\Diamond T\right] \geq \alpha$.

Theorem

The SR problem can be decided in polynomial time. Optimal pure memoryless strategies always exist and can be constructed in polynomial time.

- ▷ Linear programming (similar to SSP-E).

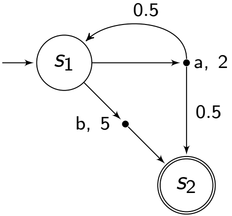
SSP-P: pseudo-PTIME algorithm (2/2)

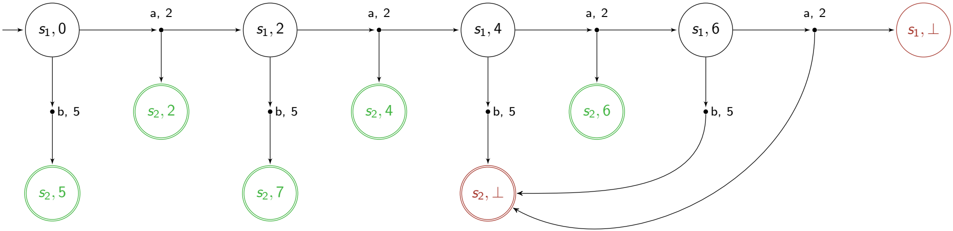


Sketch of the reduction:

1. Start from D, $T = \{s_2\}$, and $\ell = 7$.
2. Build D_ℓ by unfolding D, tracking the current sum *up to the threshold* ℓ, and integrating it in the states of the expanded MDP.

SSP-P: pseudo-PTIME algorithm (2/2)



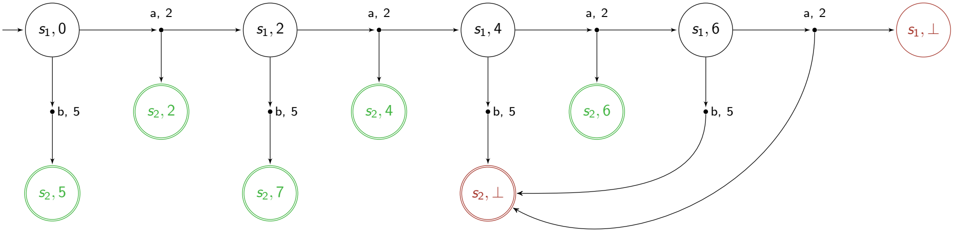


SSP-P: pseudo-PTIME algorithm (2/2)

3. Relation between runs of D and D_ℓ:

$$\mathsf{TS}^{T}(\rho) \leq \ell \quad \Leftrightarrow \quad \rho' \models \Diamond T',\ T' = T \times \{0, 1, \ldots, \ell\}.$$

4. Solve the SR problem on D_ℓ.
   - ▷ Memoryless strategy in $D_\ell \rightsquigarrow$ pseudo-polynomial memory in D in general.



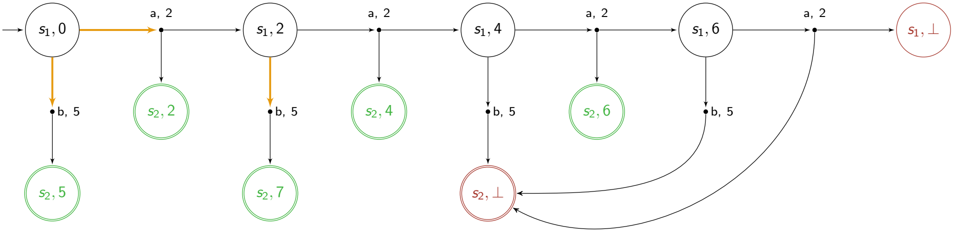
SSP-P: pseudo-PTIME algorithm (2/2)

If we just want to minimize the risk of exceeding $\ell = 7$,

- ▷ an obvious possibility is to play *b* directly,
- ▷ playing *a* only once is also acceptable.

For the SSP-P problem, **both strategies are equivalent**.

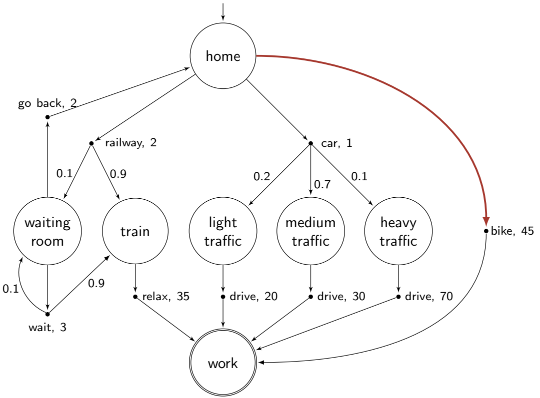
$\rightsquigarrow$ We need richer models to discriminate them!



Related work (non-exhaustive)

- SSP-P problem [Oht04, SO13].
- *Quantile queries* [UB13]: minimizing the value ℓ of an SSP-P problem for some fixed α. Recently extended to *cost problems* [HK15].
- SSP-E problem in **multi-dimensional** MDPs [FKN$^+$11].

SP-G: strict worst-case guarantees

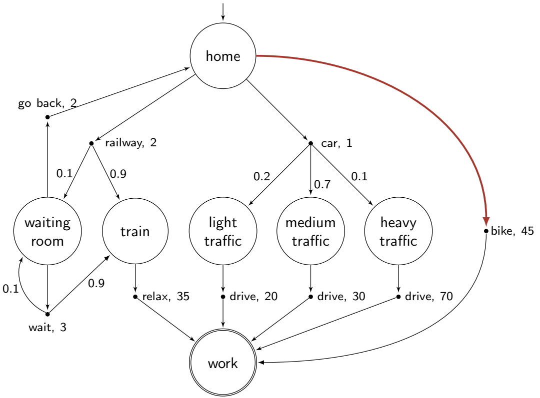


Specification: *guarantee* that work is reached within 60 minutes (to avoid missing an important meeting).

Sample strategy: take the **bike** $\rightsquigarrow \forall\, \rho \in \mathrm{Out}^{\sigma}_{D}\colon\ \mathsf{TS}^{\mathsf{work}}(\rho) \leq 60$.

Bad choices: train ($wc = \infty$) and car ($wc = 71$).

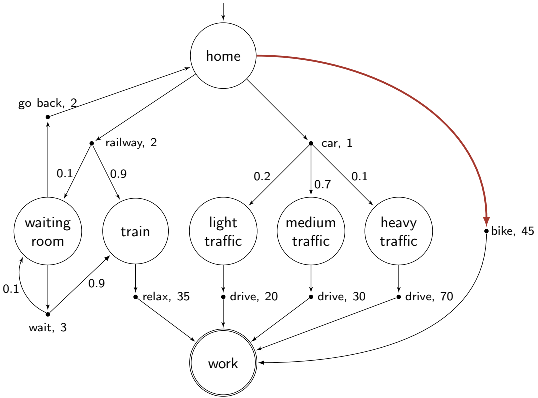
SP-G: strict worst-case guarantees



Winning surely (worst-case) $\neq$ almost-surely (proba. 1).

- ▷ Train ensures reaching work with probability one, but does not prevent runs where work is never reached.

SP-G: strict worst-case guarantees



Worst-case analysis $\rightsquigarrow$ **two-player game** against an antagonistic adversary.

- ▷ Forget about probabilities and give the choice of transitions to the adversary.

SP-G: shortest path game problem

SP-G problem

Given MDP $D = (S, s_{\mathsf{init}}, A, \delta, w)$, target set T and threshold $\ell \in \mathbb{N}$, decide if there exists a strategy σ such that for all $\rho \in \mathsf{Out}^{\sigma}_{D}$, we have that $\mathsf{TS}^{T}(\rho) \leq \ell$.

Theorem [KBB+08]

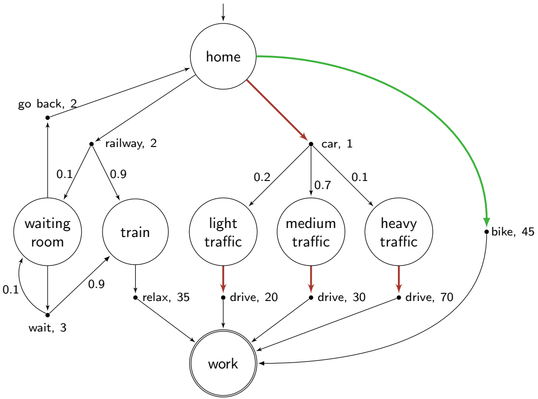
The SP-G problem can be decided in polynomial time. Optimal pure memoryless strategies always exist and can be constructed in polynomial time.

- Dynamic programming.

Related work (non-exhaustive)

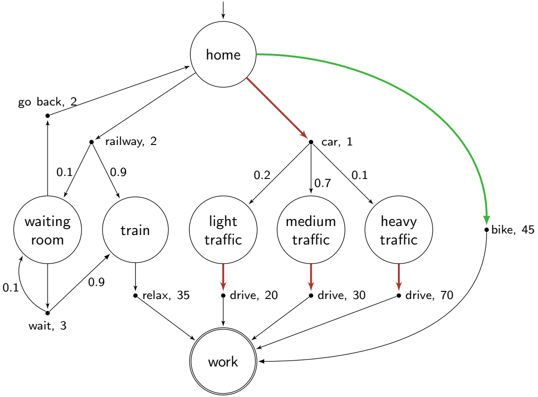
- Pseudo-PTIME for arbitrary weights [BGHM17, FGR15].
- Arbitrary weights + multiple dimensions $\rightsquigarrow$ undecidable (by adapting the proof of [CDRR15] for total-payoff).

SSP-WE = SP-G ∩ SSP-E - illustration



- SSP-E: car $\rightsquigarrow \mathbb{E} = 33$ but $wc = 71 > 60$
- SP-G: bike $\rightsquigarrow wc = 45 < 60$ but $\mathbb{E} = 45 >>> 33$

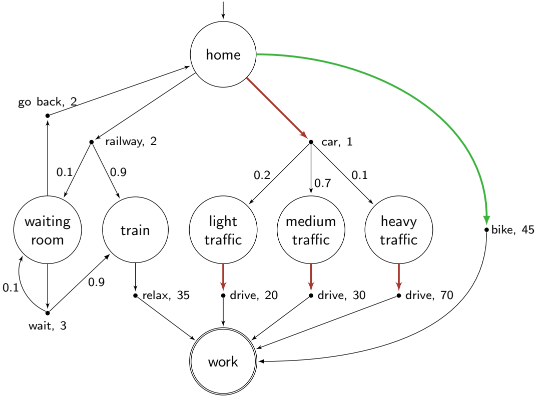
SSP-WE = SP-G ∩ SSP-E - illustration



Can we do better?

- ▷ **Beyond worst-case synthesis** [BFRR17]: minimize the expected time under the worst-case constraint.

SSP-WE = SP-G ∩ SSP-E - illustration



Sample strategy: try train up to 3 delays then switch to bike.

- $\rightsquigarrow$ $wc = 58 < 60$ and $\mathbb{E} \approx 37.34 << 45$
- $\rightsquigarrow$ pure *finite-memory* strategy

SSP-WE: beyond worst-case synthesis

SSP-WE problem

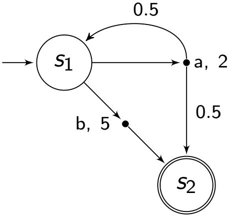
Given MDP $D = (S, s_{\mathsf{init}}, A, \delta, w)$, target set T, and thresholds $\ell_1 \in \mathbb{N}$, $\ell_2 \in \mathbb{Q}$, decide if there exists a strategy σ such that:

1. $\forall \rho \in \mathsf{Out}_D^\sigma\colon\ \mathsf{TS}^T(\rho) \leq \ell_1$,
2. $\mathbb{E}_D^\sigma(\mathsf{TS}^T) \leq \ell_2$.

Theorem [BFRR17]

The SSP-WE problem can be decided in pseudo-polynomial time and is NP-hard. Pure pseudo-polynomial-memory strategies are always sufficient and in general necessary, and satisfying strategies can be constructed in pseudo-polynomial time.

SSP-WE: pseudo-PTIME algorithm

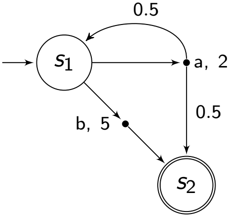


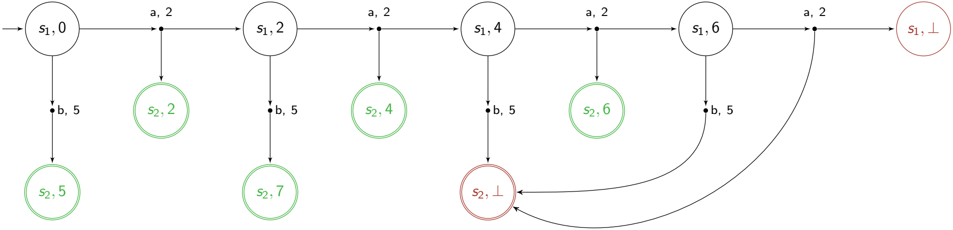
Consider SSP-WE problem for $\ell_1 = 7$ (*wc*), $\ell_2 = 4.8$ ($\mathbb{E}$).

▷ Reduction to the SSP-E problem on a pseudo-polynomial-size expanded MDP.

1. Build unfolding as for SSP-P problem w.r.t. worst-case threshold ℓ_1.

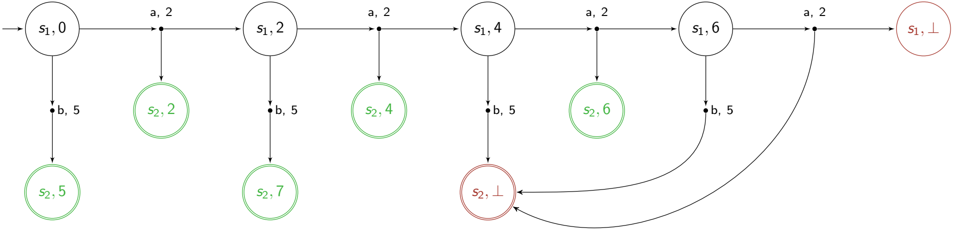
SSP-WE: pseudo-PTIME algorithm





SSP-WE: pseudo-PTIME algorithm

2. Compute R, the attractor of $T' = T \times \{0, 1, \ldots, \ell_1\}$.
3. Restrict MDP to $D' = D_{\ell_1} \downharpoonright R$, the *safe* part w.r.t. SP-G.



SSP-WE: pseudo-PTIME algorithm

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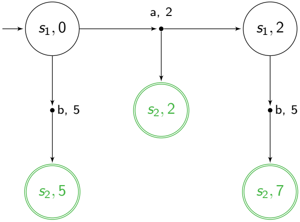
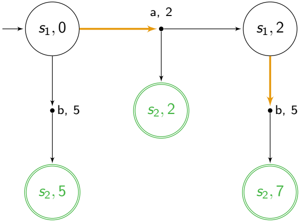


SSP-WE: pseudo-PTIME algorithm

4. Compute memoryless optimal strategy σ in D' for SSP-E.
5. Answer is $\textsc{Yes}$ iff $\mathbb{E}^{\sigma}_{D'}(\mathsf{TS}^{T'}) \leq \ell_2$.



Here,
$\mathbb{E}^{\sigma}_{D'}(\mathsf{TS}^{T'}) = 9/2.$

SSP-WE: wrap-up

SSP	complexity	strategy
SSP-E	PTIME	pure memoryless
SSP-P	pseudo-PTIME / PSPACE-h.	pure pseudo-poly.
SSP-G	PTIME	pure memoryless
SSP-WE	pseudo-PTIME / NP-h.	pure pseudo-poly.

- ▷ NP-hardness ⇒ inherently harder than SSP-E and SSP-G.

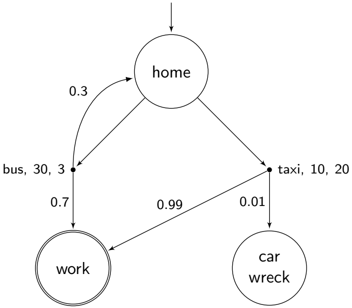
Related work (non-exhaustive)

- BWC synthesis problems for mean-payoff [BFRR17] and parity [BRR17] belong to NP ∩ coNP. Much more involved technically.

  $\Longrightarrow$ Additional modeling power for free w.r.t. worst-case problems.

- Multi-dimensional extension for mean-payoff [CR15].
- Integration of BWC concepts in UPPAAL [DJL+14].
- Optimizing the expected mean-payoff under energy constraints [BKN16] or Boolean constraints [AKV16].

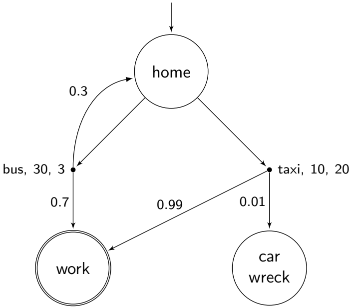
Multiple objectives $\implies$ trade-offs



Two-dimensional weights on actions: *time* and *cost*.

Often necessary to consider trade-offs: e.g., between the probability to reach work in due time and the risks of an expensive journey.

Multiple objectives $\implies$ trade-offs

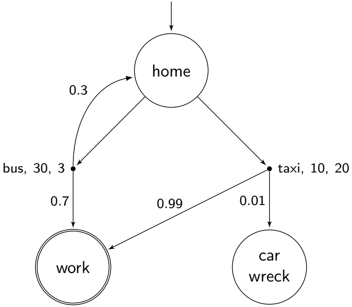


SSP-P problem considers a **single percentile constraint**.

- **C1**: 80% of runs reach work in at most 40 minutes.
  - ▷ Taxi $\rightsquigarrow \leq 10$ minutes with probability $0.99 > 0.8$.
- **C2**: 50% of them cost at most 10$ to reach work.
  - ▷ Bus $\rightsquigarrow \geq 70\%$ of the runs reach work for 3$.

Taxi $\not\models$ C2, bus $\not\models$ C1. What if we want C1 $\wedge$ C2?

Multiple objectives $\Longrightarrow$ trade-offs

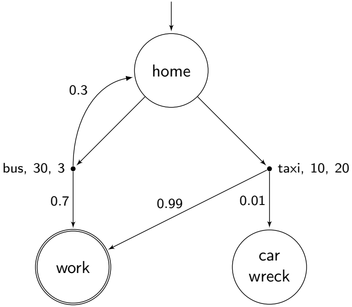


- **C1**: 80% of runs reach work in at most 40 minutes.
- **C2**: 50% of them cost at most 10$ to reach work.

Study of **multi-constraint percentile queries** [RRS17].

- ▷ Sample strategy: bus once, then taxi. Requires *memory*.
- ▷ Another strategy: bus with probability 3/5, taxi with probability 2/5. Requires *randomness*.

Multiple objectives $\implies$ trade-offs



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Study of **multi-constraint percentile queries** [RRS17].

In general, *both* memory *and* randomness are required.

$\neq$ **Previous problems.**

SSP-PQ: multi-constraint percentile queries (1/2)

SSP-PQ problem

Given d-dimensional MDP $D = (S, s_{\text{init}}, A, \delta, w)$, and $q \in \mathbb{N}$ percentile constraints described by target sets $T_i \subseteq S$, dimensions $k_i \in \{1, \ldots, d\}$, value thresholds $\ell_i \in \mathbb{N}$ and probability thresholds $\alpha_i \in [0,1] \cap \mathbb{Q}$, where $i \in \{1, \ldots, q\}$, decide if there exists a strategy σ such that query $\mathcal{Q}$ holds, with

$$\mathcal{Q} := \bigwedge_{i=1}^{q} \mathbb{P}^{\sigma}_{D}\left[\mathsf{TS}^{T_i}_{k_i} \leq \ell_i\right] \geq \alpha_i,$$

where $\mathsf{TS}^{T_i}_{k_i}$ denotes the truncated sum on dimension k_i and w.r.t. target set T_i.

Very general framework: multiple constraints related to $\neq$ dimensions, and $\neq$ target sets $\implies$ great flexibility in modeling.

SSP-PQ: multi-constraint percentile queries (2/2)

Theorem [RRS17]

The SSP-PQ problem can be decided in

- exponential time in general,
- pseudo-polynomial time for single-dimension single-target multi-contraint queries.

It is PSPACE-hard even for single-constraint queries. Randomized exponential-memory strategies are always sufficient and in general necessary, and satisfying strategies can be constructed in exponential time.

▷ Unfolding + multiple reachability problem [EKVY08, RRS17].

▷ PSPACE-hardness already true for SSP-P [HK15].

$\rightsquigarrow$ SSP-PQ = wide extension for basically no price in complexity.

SSP-PQ: wrap-up

SSP	complexity	strategy
SSP-E	PTIME	pure memoryless
SSP-P	pseudo-PTIME / PSPACE-h.	pure pseudo-poly.
SSP-G	PTIME	pure memoryless
SSP-WE	pseudo-PTIME / NP-h.	pure pseudo-poly.
SSP-PQ	EXPTIME (p.-PTIME) / PSPACE-h.	randomized exponential

- ▷ SSP-PQ is undecidable for arbitrary weights in multi-dimensional MDPs, even with a unique target set [RRS17].

Percentile queries: overview (1/2)

- **Wide range of payoff functions**
  - multiple reachability,
  - inf, sup, lim inf, lim sup,
  - mean-payoff ($\overline{\mathsf{MP}}$, $\underline{\mathsf{MP}}$),
  - shortest path (SP),
  - discounted sum (DS).
- **Several variants**:
  - multi-dim. multi-constraint,
  - single-dim. multi-constraint,
  - single-constraint.
- For each one:
  - algorithms,
  - lower bounds,
  - memory requirements.

$\rightsquigarrow$ **Complete picture** for this new framework.

Percentile queries: overview (2/2)

	Single-constraint	Single-dim. Multi-constraint	Multi-dim. Multi-constraint
Reachability	P [Put94]	$\mathsf{P}(D){\cdot}\mathsf{E}(\mathcal{Q})$ [EKVY08], PSPACE-h	—
$f \in \mathcal{F}$	P [CH09]	P	$\mathsf{P}(D){\cdot}\mathsf{E}(\mathcal{Q})$ PSPACE-h.
$\overline{\mathsf{MP}}$	P [Put94]	P	P
$\underline{\mathsf{MP}}$	P [Put94]	$\mathsf{P}(D){\cdot}\mathsf{E}(\mathcal{Q})$	$\mathsf{P}(D){\cdot}\mathsf{E}(\mathcal{Q})$
SP	$\mathsf{P}(D){\cdot}\mathsf{P}_{ps}(\mathcal{Q})$ [HK15] PSPACE-h. [HK15]	$\mathsf{P}(D){\cdot}\mathsf{P}_{ps}(\mathcal{Q})$ (one target) PSPACE-h. [HK15]	$\mathsf{P}(D){\cdot}\mathsf{E}(\mathcal{Q})$ PSPACE-h. [HK15]
ε-gap DS	$\mathsf{P}_{ps}(D, \mathcal{Q}, \varepsilon)$ NP-h.	$\mathsf{P}_{ps}(D, \varepsilon){\cdot}\mathsf{E}(\mathcal{Q})$ NP-h.	$\mathsf{P}_{ps}(D, \varepsilon){\cdot}\mathsf{E}(\mathcal{Q})$ PSPACE-h.

- $\mathcal{F} = \{\inf, \sup, \liminf, \limsup\}$
- D = model size, $\mathcal{Q}$ = query size
- $\mathsf{P}(x)$, $\mathsf{E}(x)$ and $\mathsf{P}_{ps}(x)$ resp. denote polynomial, exponential and pseudo-polynomial time in parameter x.

All results without reference are established in [RRS17].

Percentile queries: overview (2/2)

	Single-constraint	Single-dim. Multi-constraint	Multi-dim. Multi-constraint
Reachability	P [Put94]	P(D)·E($\mathcal{Q}$) [EKVY08], PSPACE-h	—
$f \in \mathcal{F}$	P [CH09]	P	P(D)·E($\mathcal{Q}$) PSPACE-h.
$\overline{\mathsf{MP}}$	P [Put94]	P	P
$\underline{\mathsf{MP}}$	P [Put94]	P(D)·E($\mathcal{Q}$)	P(D)·E($\mathcal{Q}$)
SP	P(D)·P_{ps}($\mathcal{Q}$) [HK15] PSPACE-h. [HK15]	P(D)·P_{ps}($\mathcal{Q}$) (one target) PSPACE-h. [HK15]	P(D)·E($\mathcal{Q}$) PSPACE-h. [HK15]
ε-gap DS	$\mathsf{P}_{ps}(D, \mathcal{Q}, \varepsilon)$ NP-h.	$\mathsf{P}_{ps}(D, \varepsilon)$·E($\mathcal{Q}$) NP-h.	$\mathsf{P}_{ps}(D, \varepsilon)$·E($\mathcal{Q}$) PSPACE-h.

In most cases, only **polynomial in the model size**.

- ▷ In practice, the query size can often be bounded while the model can be very large.

Summary: stochastic shortest path problem

- **SSP-E:** minimize the expected sum to target.
  - ▷ Actual outcomes may vary greatly.
- **SSP-P:** maximize the probability of acceptable performance.
  - ▷ No control over the quality of bad runs, no average-case performance.
- **SP-G:** maximize the worst-case performance, extreme risk-aversion.
  - ▷ Strict worst-case guarantees, no average-case performance.
- **SSP-WE:** SSP-E $\cap$ SP-G.
  - ▷ Based on beyond worst-case synthesis [BFRR17].
- **SSP-PQ:** extends SSP-P to multi-constraint percentile queries [RRS17].
  - ▷ Multi-dimensional, flexible, trade-offs.
  - ▷ Complexity usually acceptable w.r.t. model size.

Thank you! Any question?

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SP-G: PTIME algorithm

1. Cycles are bad $\implies$ must reach target within $n = |S|$ steps.
2. $\forall\, s \in S$, $\forall\, i$, $0 \leq i \leq n$, compute $\mathbb{C}(s, i)$.
   - ▷ Lowest bound on cost to T from s that we can ensure in i steps.
   - ▷ **Dynamic programming** (polynomial time).

   Initialize

   $$\forall\, s \in T,\ \mathbb{C}(s, 0) = 0, \qquad \forall\, s \in S \setminus T,\ \mathbb{C}(s, 0) = \infty.$$

   Then, $\forall\, s \in S$, $\forall\, i$, $1 \leq i \leq n$,

   $$\mathbb{C}(s, i) = \min\left[\mathbb{C}(s, i-1), \min_{a \in A(s)} \max_{s' \in \mathrm{Supp}(\delta(s,a))} w(a) + \mathbb{C}(s', i-1)\right].$$

3. Winning strategy iff $\mathbb{C}(s_{\mathrm{init}}, n) \leq \ell$.

SSP-PQ: EXPTIME / pseudo-PTIME algorithm

1. Build an unfolded MDP D_ℓ similar to SSP-P case:
   - ▷ stop unfolding when *all* dimensions reach sum $\ell = \max_i \ell_i$.
2. Maintain *single*-exponential size by defining an equivalence relation between states of D_ℓ:
   - ▷ $S_\ell \subseteq S \times (\{0, \ldots, \ell\} \cup \{\bot\})^d$,
   - ▷ pseudo-poly. if $d = 1$.
3. For each constraint i, compute a target set R_i in D_ℓ:
   - ▷ $\rho \models$ constraint i in $D \iff \rho' \models \Diamond R_i$ in D_ℓ.
4. Solve a **multiple reachability problem** on D_ℓ.
   - ▷ Generalizes the SR problem [EKVY08, RRS17].
   - ▷ Time polynomial in $|D_\ell|$ but exponential in q.
   - ▷ Single-dim. single target queries ⇒ absorbing targets ⇒ polynomial-time algorithm.